Chirped Pulses and the Meaning of Fourier Transform Limited

One of the biggest problems associated with ultrashort pulse lasers stems from the fact that in many cases, characterizing the pulse shape is more difficult than generating the pulses. Most researchers would like to know the functional form of the complex electric field envelope, $E(t)$, or even the intensity, $I(t)$, but current technology is not capable of providing the bandwidth necessary to directly perform these measurements.

Although the past few years have seen the development of several techniques that allow iterative reconstruction of the pulse shape [1-6], the most common techniques used to determine pulse shape information are autocorrelations (intensity or interferometric) or spectral measurements. Autocorrelations suffer from the fact that the autocorrelation function, not the pulse shape is measured and the shape of the resulting autocorrelation is strongly dependent on the pulse shape. Even a relatively simple pulse parameter such as Full Width Half Maximum can vary greatly depending on the assumed pulse shape.

Spectral measurements of ultrashort pulses can also be used to deduce some information about the temporal intensity profile. This is because the spectral intensity, $I(\lambda)$, (or more correctly, $I(n)$, the frequency dependent intensity) is related to its temporal intensity, $I(t)$, by the Fourier Transform. The problem with this approach is that spectral measurements are only capable of providing the intensity, $I(\lambda)$, thus ignoring any frequency dependent phase that may be present. This frequency dependent phase results in a pulse that has a finite amount of "chirp."

If a pulse is chirp free, the inverse Fourier transform of the square root of the spectral intensity provides an accurate representation of the pulse as shown in the equation below. This equation serves as the definition of a "transform limited" pulse independent of the actual shape of the pulse. If there is some chirp in the pulse, this procedure is not valid.

$$I(t) = (F^{-1}\{I(n)^{1/2}\})^2$$

(only valid when pulse is transform limited)

Unfortunately, there is no direct procedure for determining if a pulse is transform limited. One can deduce the degree to which a pulse is transform limited but only at the risk of assuming the functional form of the temporal pulse shape. This is done by comparing the measured time-bandwidth product (TBWP) to that calculated by theory. The conventional definition of the time-bandwidth product is shown below where Dn_{FWHM} and Dt_{FWHM} are the Full Width Half Maximum of $I(n)$ and $I(t)$ respectively.

$$TBWP = Dn_{FWHM} \, Dt_{FWHM}$$

Note that, as in the case of the autocorrelation function, the time-bandwidth product is strongly dependent on the functional form of the pulse shape (see below). A further complication is the fact that the Dt_{FWHM} is typically calculated from an autocorrelation measurement which is also dependent on the assumed functional form of the pulse shape.

Gaussian	0.44
Hyperbolic Secant Squared	0.31
Square	0.89
Triangle	0.54
Lorentzian	0.22

To illustrate the issues presented previously, the figures on the opposite page show the spectral intensity and the corresponding pulse shape of three different pulses. Figure 1 shows a pure 100 fs Gaussian pulse at 800 nm that is transform limited. The time-bandwidth product of this pulse is 0.44. Figure 2 shows the spectrum of a transform limited Gaussian pulse where the spectrum below 794 nm has been clipped. This clipping can occur, for example, if the stretcher or compressor of a chirped pulse amplifier system has been misaligned. The corresponding pulse shape in the temporal domain is not the same as the pulse in Figure 1 even though both pulses are transform limited. The pulsewidth is now 125 fs and the corresponding time-bandwidth product is 0.55 which is not the same value as in Figure 1. This is a good illustration of the problems that can arise when the time-bandwidth product is the only measurement used to determine whether a pulse is transform limited. As a final example, the pulse in Figure 3 is a Gaussian with a non-zero amount of frequency dependent phase added which can arise from a mismatch between the stretcher and the compressor. The resultant time-bandwidth product (0.55) is the same as the clipped Gaussian of Figure 2 even though the pulse in Figure 3 is not transform limited.

In summary, if it is important to check that your pulse is truly chirp free, simply measuring the TBP can lead to inconclusive and possibly misleading results.

If any questions arise during your research, don't hesitate to contact Positive Light for expert technical assistance in all branches of ultrafast laser technology.

References

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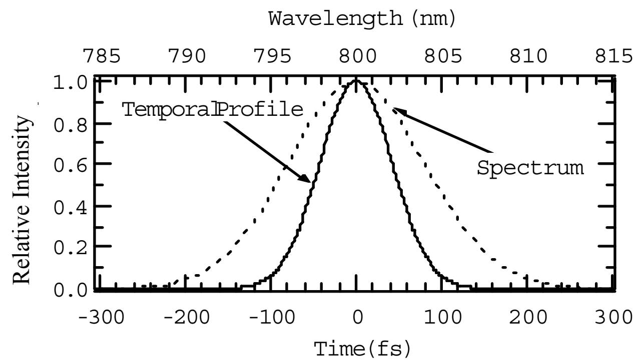

Figure 1: Spectral and temporal profile of a 100 fs, transform limited Gaussian pulse. The $Dn_{FWHM} Dt_{FWHM}$ = 0.44 [6]. The dotted curve is the spectrum, $I(\lambda)$, and the solid curve is the temporal profile, $I(t)$.

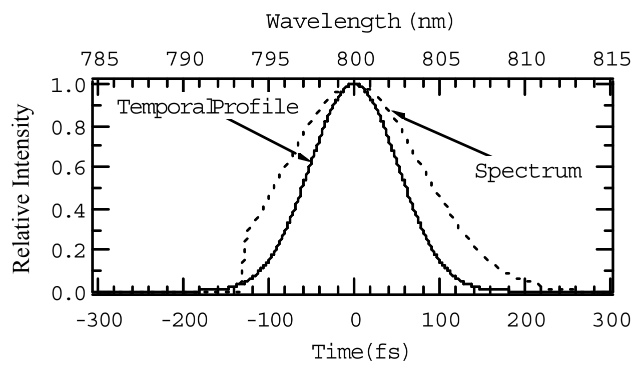

Figure 2: Spectral and temporal profile of a Gaussian pulse with the spectrum clipped below 794nm. The $Dn_{FWHM} Dt_{FWHM}$ = 0.55 even though the pulse is transform limited. Note the broadened pulsewidth (125 fs) as compared to the pulsewidth in Figure 1.

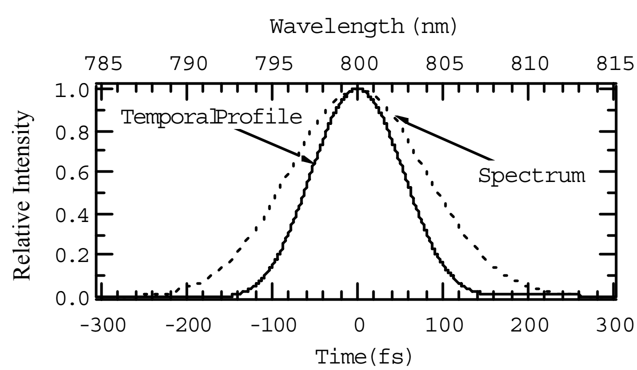

Figure 3: Spectral and temporal profile of a Gaussian pulse with the same spectral width as Figure 1 but with a finite amount of spectral phase added. Even though the $Dn_{FWHM} Dt_{FWHM}$ = 0.55 (as in Figure 2), this pulse is NOT transform limited.

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